quency in radians, and ω_{SP} is the short-period frequency in rad/sec.

For most of the configurations simulated a nondimensional delay parameter can be defined in either of the following two ways:

$$\text{Delay parameter} = a/P_E = a/(2\pi/\omega_E) \quad (4)$$

$$\text{Delay parameter} = a/P_{SP} = (\phi_{CS}/\omega_{SP})/(2\pi/\omega_{SP}) = \phi_{CS}/2\pi \quad (5)$$

In Eq. (4), P_E is the equivalent undamped period of the equivalent second-order system. In Eq. (5), P_{SP} is the undamped period of the airplane short period. Equations (4) and (5) are essentially equivalent only when the break point frequency of the control system elements are sufficiently higher than the airplane short-period frequency.

The figures presented in the paper demonstrate that a strong correlation exists between pilot ratings, PIO ratings, and the delay parameter as defined by Eqs. (4) and (5) for all the configurations evaluated. With a degradation in control system dynamics, the delay parameter increases, PIO tendencies increase, and a deterioration in handling qualities occurs. With a delay parameter between 0.2 and 0.26, the control system phase shift increases to between 70° and 90°. The pilot rating becomes 10, the PIO rating becomes 6, and the airplane is considered uncontrollable because of divergent PIO tendencies.

A comparison is made of fixed-base ground simulator vs flight evaluations and this comparison indicates that configurations with significant PIO tendencies are rated poorer in flight than on the ground. In evaluating PIO tendencies, ground simulator results are not conservative and can be very misleading.

Higher-Order Control System Dynamics and Longitudinal Handling Qualities

DANTE A. DIFRANCO*
Cornell Aeronautical Laboratory, Buffalo, N. Y.

An experimental investigation of the effects of higher-order control system dynamics on the longitudinal handling qualities of a fighter airplane are discussed. This research was undertaken using the USAF/CAL variable stability T-33 airplane as an in-flight simulator. Different higher-order responses were simulated by altering the elevator feel system, elevator actuator, and airplane short-period characteristics. Essentially the same configurations were evaluated by two pilots using a revised pilot rating scale. One pilot also rated the configurations for their pilot-induced oscillations (PIO) tendencies. Comments and ratings were related to a response delay parameter. Many of the higher-order control systems investigated produced pronounced PIO tendencies in flight, and some were considered unflyable with certain higher-order characteristics. A comparison of fixed-base and in-flight evaluations indicated that configurations with significant PIO tendencies were rated poorer in flight, and configurations with little or no PIO tendencies were rated better in flight.

Nomenclature

a_i = system delay time ($i = LF, \phi$), sec
f_i = frequency ($i = a, FS_1, FS_2$), cps
F_i = control force ($i = ES$), lb
g = acceleration of gravity, ft/sec²
$K_1, K_2 \ldots K_n$ = characteristic equation constants
L_α = dimensional lift force derivative with angle of attack, 1/sec
$M_{\delta a}$ = dimensional pitching moment derivative with respect to elevator deflection, 1/sec²
n_z = normal acceleration, g's
n = number of roots or order of characteristic equation
P_i = period ($i = E, SP$), sec
$PIOR$ = pilot-induced oscillation rating
s = Laplace operator
V = airplane velocity, fps
α = airplane angle of attack from trim level flight, rad
δ_i = control surface or control stick deflection ($i = e, ES$), rad or in.
ζ_a = damping ratio ($i = a, a_1, a_2, E, FS, FS_1, FS_2, p, SP$)
θ = airplane pitch angle, rad
$\theta(\text{eta})_\epsilon$ = pitch angle tracking error, deg
ϕ_a = phase angle of control system evaluated at airplane short-period frequency, rad or deg
ω_i = undamped natural frequency ($i = a, a_1, a_2, E, FS, FS_1, FS_2, p, SP$), rad/sec
$(\dot{\ })$ = first derivative with respect to time

Subscripts

a, a_1, a_2 = actuator dynamics
e = elevator surface
E = equivalent
ES = elevator stick
FS, FS_1, FS_2 = feel system dynamics
LF = match of lowest-order coefficients in characteristic equation
0 = initial condition
p = airplane phugoid
SS = steady-state value
SP = airplane short period
ϕ = value determined from control system phase shift at airplane short-period frequency

Received July 11, 1969; presented as Paper 69-768 at the AIAA Aircraft Design and Operations Meeting, Los Angeles, Calif., July 14–16, 1969; revision received November 10, 1969. The paper is based on research performed for the Air Force Flight Dynamics Laboratory under Contract AF 33(615)-3294. W. Smith was project engineer for the Flight Dynamics Laboratory.

* Aeronautical Research Engineer, Flight Research Department. Member AIAA.

I. Introduction

IT has been known for some time that control system dynamics, as well as open-loop airplane dynamics, affect the handling qualities of airplanes. Most handling qualities investigations have been concerned with the effects of variations in certain open-loop airplane parameters. The feel system and elevator actuator characteristics have, in general, been held constant. In many cases, the dynamic characteristics of the control system, and the effects of these characteristics on the pilot-airplane closed-loop response, have not been adequately documented or systematically investigated.

An investigation of higher-order control system dynamics is of special interest since modern high performance fighters employ flight control systems that increase the order of the airplane responses to pilot stick inputs. These responses can become very "nonairplane-like" to the pilot, and how they will affect the airplane handling qualities is not clearly understood.

Although some of the results of this flight test program are also applicable to simple and less sophisticated control systems with simple time lags, the program is considered to be only an introductory one in a systematic in-flight investigation of the handling qualities of airplanes with higher-order control system dynamics. This investigation is based on a similar ground simulator program.[1] The higher-order characteristics introduced by modern control systems are a result of the same elements (feel system, actuator) investigated here, but different higher-order responses can result from the addition of feedback loops. Such loops may consist of airplane responses that generate inputs to the elevator actuator, a bobweight that generates an input to the stick, etc. Control systems with these feedback loops are not considered in this paper, but some of these systems are presently under investigation.

II. In-Flight Simulation

This in-flight investigation was conducted in the USAF/CAL variable stability T-33 airplane. The control system of the T-33 consists of an elevator feel system and an elevator actuator.[2] Higher-order response characteristics were simulated by altering the elevator stick feel system and the elevator actuator dynamics in conjunction with four different sets of longitudinal short-period airplane dynamics. The dynamics of any of the three elements (feel system, actuator, and airplane) could be changed independently of the others. Airplane dynamics were simulated using the response feedback system of the variable stability T-33 airplane.[2]

A block diagram of the pilot in-flight closed-loop control is shown as Fig. 1. Control is initiated by the pilot by applying a control force (F_{ES}). Through the dynamics of the simulated elevator feel system, this force results in a stick displacement (δ_{ES}) which is an input to the simulated elevator actuator. The elevator response (δ_e) is a function of the actuator dynamics, and commands the response of the airplane as determined by the simulated airplane dynamics.

A configuration matrix, established for in-flight evaluation, is shown as Table 1. The three sets of feel system dynamics (fast, medium, and slow) simulated were well represented by a fourth-order transfer function of the following form:

$$\frac{\delta_{ES}(s)}{F_{ES}(s)} = \frac{\omega_{FS_1}{}^2\omega_{FS_2}{}^2[(\delta_{ES}/F_{ES})_{SS}]}{(s^2 + 2\zeta_{FS_1}\omega_{FS_1}s + \omega_{FS_1}{}^2)\,(s^2 + 2\zeta_{FS_2}\omega_{FS_2}s + \omega_{FS_2}{}^2)} \quad (1)$$

The undamped frequencies and damping ratios associated with each of the feel systems simulated are shown in Table 2. Since reasonable separation exists between low- and high-frequency roots of each of the feel systems, the feel system responds essentially as a second-order system at the lower frequency with a slight delay introduced by the higher-frequency roots. All the feel systems are satisfactorily damped.

Both the order and frequency of the actuator were varied as indicated in Table 1. The second-, fourth-, and fifth-order actuator transfer functions, respectively, can be represented as follows:

$$\frac{\delta_e(s)}{\delta_{ES}(s)} = \frac{(\omega_a{}^2)(\delta_e/\delta_{ES})_{SS}}{s^2 + 2\zeta_a\omega_a s + \omega_a{}^2} \quad (2)$$

$$\frac{\delta_e(s)}{\delta_{ES}(s)} = \frac{(\omega_a{}^4)(\delta_e/\delta_{ES})_{SS}}{(s^2 + 2\zeta_{a_1}\omega_a s + \omega_a{}^2)(s^2 + 2\zeta_{a_2}\omega_a s + \omega_a{}^2)} \quad (3)$$

$$\frac{\delta_e(s)}{\delta_{ES}(s)} = \frac{(\omega_a{}^5)(\delta_e/\delta_{ES})_{SS}}{(s + \omega_a)(s^2 + 2\zeta_{a_1}\omega_a s + \omega_a{}^2)(s^2 + 2\zeta_{a_2}\omega_a s + \omega_a{}^2)} \quad (4)$$

The simulated actuator undamped frequencies are shown in Table 1. The damping ratios for each of the actuators were determined by a Butterworth distribution of actuator roots. For the second-order actuator, $\zeta_{a_1} = 0.707$; for the fourth-order actuator, $\zeta_{a_1} = 0.925$ and $\zeta_{a_2} = 0.383$; and for the fifth-order actuator, $\zeta_{a_1} = 0.810$ and $\zeta_{a_2} = 0.309$.

Three sets of longitudinal short-period characteristics were specified for a fighter in "up-and-away" flight (A, B, and C), and one set was specified for landing approach (LA). The airplane undamped frequencies and damping ratios are indicated in Table 1. The airplane angle of attack, pitch rate,

Table 1 Configuration matrix established for in-flight evaluation[c]

Feel system dynamics	Actuator dynamics		Airplane dynamics: Up-and-away configurations A	B	C	Landing approach LA
	Order	f_a, cps	ω_{SP} = 2.7 rad/sec, ζ_{SP} = 0.55	ω_{SP} = 2.7 rad/sec, ζ_{SP} = 0.24	ω_{SP} = 5.1 rad/sec, ζ_{SP} = 0.43	ω_{SP} = 2.0 rad/sec, ζ_{SP} = 0.56
Fast		10.0	●[a] A(F)-2(10)√[b]	● B(F)-2(10)√	● C(F)-2(10)√	● LA(F)-2-(10)√
	2	2.5	● A(F)-2(2.5)√	B(F)-2(2.5)√	C(F)-2(2.5)	LA(F)-2(2.5)√
		1.0	● A(F)-2(1)√	● B(F)-2(1)√	● C(F)-2(1)√	● LA(F)-2(1)√
	4	2.5	● A(F)-4(2.5)√	● B(F)-4(2.5)√	● C(F)-4(2.5)√	LA(F)-4(2.5)√
	5	2.5	● A(F)-5(2.5)√	● B(F)-5(2.5)√	C(F)-5(2.5)√	LA(F)-5(2.5)√
		1.0	A(F)-5(1)√	B(F)-5(1)√	C(F)-5(1)√	● LA(F)-5(1)√
Medium	2	10.0	● A(M)-2(10)√	B(M)-2(10)	C(M)-2(10)√	● LA(M)-2(10)√
		2.5	A(M)-2(2.5)√	B(M)-2(2.5)	C(M)-2(2.5)	● LA(M)-2(2.5)√
Slow	2	10.0	● A(S)-2(10)√	● B(S)-2(10)√	● C(S)-2(10)√	● LA(S)-2(10)√
		2.5	A(S)-2(2.5)√	B(S)-2(2.5)	C(S)-2(2.5)	● LA(S)-2(2.5)√

[a] Configurations evaluated in fixed-base ground simulator.
[b] Configurations evaluated in flight.
[c] Numbers and letters in blocks refer to configuration designations.

Table 2 Elevator feel system characteristics

Feel system	ω_{FS_1}, rad/sec	f_{FS_1}, cps	ζ_{FS_1}	ω_{FS_2}, rad/sec	f_{FS_2}, cps	ζ_{FS_2}
Fast	30.7	4.88	0.83	60.0	9.55	0.98
Medium	9.71	1.55	0.60	49.1	7.83	0.94
Slow	5.70	0.906	0.62	49.1	7.83	0.94

and normal acceleration transfer functions take the following form:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{M_{\delta_e}}{s^2 + 2\zeta_{SP}\omega_{SP}s + \omega_{SP}^2} \tag{5}$$

$$\frac{\dot{\theta}(s)}{\delta_e(s)} = \frac{M_{\delta_e}(s + L_\alpha)}{s^2 + 2\zeta_{SP}\omega_{SP}s + \omega_{SP}^2} \tag{6}$$

$$\frac{n_z(s)}{\delta_e(s)} = \frac{(V_0/g)M_{\delta_e}L_\alpha}{s^2 + 2\zeta_{SP}\omega_{SP}s + \omega_{SP}^2} \tag{7}$$

Airplane short-period dynamics and handling qualities characteristics are not only a function of denominator parameters, but also a function of numerator parameters such as L_α, V_0, and $(n_z/\alpha)_{SS}$.[3-5] These parameters are related as follows:

$$(n_z/\alpha)_{SS} = (V_0/g)(L_\alpha) \tag{8}$$

The "up-and-away" airplane configurations were simulated and evaluated at an indicated airspeed of 250 knots at 23,000-ft pressure altitude. The landing approach simulation and evaluation were conducted at 140 knots with the landing gear down and the flaps deflected 25°. The landing approach evaluation also included an Instrument Landing System (ILS) approach.

The values of L_α and $(n_z/\alpha)_{SS}$ of the T-33 that correspond to these flight conditions are shown in Table 3. The elevator spring rates $(F_{ES}/\delta_{ES})_{SS}$ and the stick force gradients $(F_{ES}/n_z)_{SS}$ are also shown in the table. These values were considered satisfactory and representative for a fighter based on the ground simulator program[1] and preliminary in-flight evaluations.

No attempt was made to alter the longitudinal phugoid characteristics of the basic T-33 airplane. For the up-and-away flight condition, $\omega_p = 0.07$ rad/sec and $\zeta_p = 0.05$. For the landing approach flight condition, $\omega_p = 0.2$ rad/sec and $\zeta_p = 0.096$.

The higher-order response of the airplane to stick force inputs is a product of the feel system, actuator, and airplane transfer functions.

$$\alpha(s)/F_{ES}(s) = [\delta_{ES}(s)/F_{ES}(s)][\delta_e(s)/\delta_{ES}(s)][\alpha(s)/\delta_e(s)] \tag{9}$$

$$\dot{\theta}(s)/F_{ES}(s) = [\delta_{ES}(s)/F_{ES}(s)][\delta_e(s)/\delta_{ES}(s)][\dot{\theta}(s)/\delta_e(s)] \tag{10}$$

$$n_z(s)/F_{ES}(s) = [\delta_{ES}(s)/F_{ES}(s)][\delta_e(s)/\delta_{ES}(s)][n_z(s)/\delta_e(s)] \tag{11}$$

Each configuration in the matrix of Table 1 is designated by letters and numbers. The first letter refers to the airplane configuration, and the letter in parentheses designates the feel system (fast, medium, or slow). The first number in the designation identifies the actuator order; the number in parentheses is the actuator frequency in cycles per second.

Those configurations in the matrix with a check mark (√) were evaluated in flight by one or more evaluation pilots. Some of the configurations, with some slight differences, were also evaluated in a previously conducted ground simulator program.[1] These are identified by a solid dot (●). A few configurations were evaluated that are not part of the matrix of Table 1.[6] These configurations in no way altered the results or the conclusions drawn from the experiment. Some configurations, especially those with significant PIO tendencies, were also evaluated with both higher and lower stick force gradients $(F_{ES}/n_z)_{SS}$. The purpose was to assess the effects of stick force gradient on PIO tendencies and handling qualities.

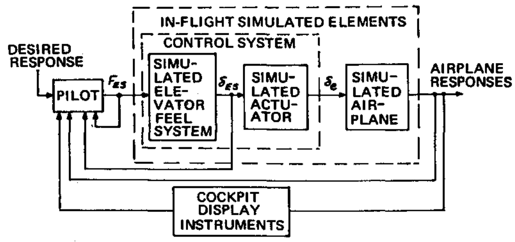

Fig. 1 Longitudinal control loops in flight.

No attempt was made to alter the basic lateral-directional dynamic stability characteristics of the T-33 airplane. The fact that few, if any, really adverse comments were directed at the lateral-directional stability and control characteristics indicates that these characteristics were satisfactory and did not influence the longitudinal evaluation.

The simulated feel system, actuator, and airplane dynamics were verified from both ground and in-flight calibration records. The response of the airplane to step stick force inputs was recorded in flight, and these records verified that the higher-order response characteristics desired were in fact simulated. Records were also obtained of tracking performance and performance during an ILS approach. Although there were some slight deviations from the average configuration characteristics presented here, none of the deviations was considered significant.[6]

III. In-Flight Evaluation

The simulated configurations were flown and evaluated by the pilot in the front seat of the USAF/CAL variable stability T-33 airplane. The control system and airplane dynamics simulated were determined by control system and variable stability gains set by the safety pilot in the rear cockpit. The evaluation pilot was not informed of the configuration to be evaluated.

Each configuration was evaluated as an all-weather fighter under both Visual Flight Rules (VFR) and Instrument Flight Rules (IFR) conditions. During the IFR evaluation, the pilot wore a cardboard hood which blocked his outside view and made concentration on instruments easier. The ILS approach was an instrument approach down to the middle marker at 300 ft above the runway. At the middle marker, the hood was removed and the approach was continued visually down to a flare a few feet above the runway, followed by a go-around.

As part of the evaluation, the pilot was asked to perform a series of fighter mission tasks both VFR and IFR. Included in the evaluation were separate altitude and attitude tracking tasks. The pilot was asked to compensate for the tracking error displayed on the all-attitude indicator. The landing approach configurations were also evaluated performing an ILS approach. Azimuth error and glide slope error were displayed by the ILS cross pointer and were also recorded in flight.

Table 3 Simulated airplane characteristics

Airplane configuration	L_α, sec^{-1}	$(n_z/\alpha)_{SS}$, g/rad	$(F_{ES}/\delta_{ES})_{SS}$, lb/in.	$(F_{ES}/n_z)_{SS}$, lb/g
A	1.25	22.5	26.3	8.0
B	1.25	22.5	26.3	8.0
C	1.25	22.5	26.3	8.0
LA	0.91	6.1	7.5	16.0

Table 4 Pilot rating scale

Controllable — Capable of being controlled or managed in context of mission with available pilot attention.	Acceptable — May have deficiencies which warrant improvement, but adequate for mission. Pilot compensation, if required to achieve acceptable performance, is feasible.	Satisfactory — Meets all requirements and expectations, good enough without improvement. Clearly adequate for mission.	Excellent, highly desirable.	A1
			Good, pleasant, well behaved.	A2
			Fair, some mildly unpleasant characteristics. Good enough for mission without improvement.	A3
		Unsatisfactory — Reluctantly acceptable. Deficiencies which warrant improvement. Performance adequate for mission with feasible pilot compensation.	Some minor but annoying deficiencies. Improvement is requested. Effect on performance is easily compensated for pilot.	A4
			Moderately objectionable deficiencies. Improvement is needed. Reasonable performance requires considerable pilot compensation.	A5
			Very objectionable deficiencies. Major improvements are needed. Requires best available pilot compensation to achieve acceptable performance.	A6
	Unacceptable — Deficiencies which require mandatory improvement. Inadequate performance for mission even with maximum feasible pilot compensation.		Major deficiencies which require mandatory improvement for acceptance. Controllable. Performance inadequate for mission, or pilot compensation required for minimum acceptable performance in mission is too high.	U7
			Controllable with difficulty. Requires substantial pilot skill and attention to retain control and continue mission.	U8
			Marginally controllable in mission. Requires maximum available pilot skill and attention to retain control.	U9
Uncontrollable — Control will be lost during some portion of the mission.			Uncontrollable in mission	10

The evaluation pilot was asked to make specific comments on each configuration evaluated, and these comments were recorded in flight. As part of the comments, the evaluation pilot was asked to give each configuration a pilot rating and PIO rating based on the rating scales shown as Tables 4 and 5. This pilot rating scale was devised to overcome the difficulties experienced with previous rating scales.[7] The new scale is clearly mission oriented. It is arranged so that the pilot can make a series of sequential decisions in arriving at a rating. This scale also provides better word descriptions associated with each rating category. The PIO rating scale proved successful in PIO evaluations on another flight program.[5]

Two experienced evaluation pilots were used in this flight test program, pilot B and pilot H. They are the same pilots who participated in the fixed-base ground simulator program.[1]

Table 5 PIO tendency rating scale

Description	Numerical rating
No tendency for pilot to induce undesirable motions	1
Undesirable motions tend to occur when pilot initiates abrupt maneuvers or attempts tight control. These motions can be prevented or eliminated by pilot technique.	2
Undesirable motions easily induced when pilot initiates abrupt maneuvers or attempts tight control. These motions can be prevented or eliminated but only at sacrifice to task performance or through considerable pilot attention and effort.	3
Oscillations tend to develop when pilot initiates abrupt maneuvers or attempts tight control. Pilot must reduce gain or abandon task to recover.	4
Divergent oscillations tend to develop when pilot initiates abrupt maneuvers or attempts tight control. Pilot must open loop by releasing or freezing the stick.	5
Disturbance or normal pilot control may cause divergent oscillation. Pilot must open control loop by releasing or freezing the stick.	6

IV. Summary of Pilot Comments

Airplane A, with a fast feel system and actuator, Configuration A(F)-2(10), was considered reasonably satisfactory by both pilots. As the order was increased and the breakpoint frequency reduced for the control system elements (Table 1), the most frequent comment was on the amount of delay or lag in the airplane response following a stick force input. The rapidity of the response after the delay was also considered to be a factor in the closed-loop control difficulties. With the delay, the airplane response was difficult to predict, and there was a tendency to overdrive the airplane and pump the stick to compensate for the delay.

The pilots complained first of "bobble" tendencies, overcontrol, and a lack of precise attitude and g control. As the delay increased, precise control deteriorated further, tracking performance became poorer and then impossible, and PIO tendencies developed and then became divergent. With the poorest configurations, PIO tendencies developed just trying to fly the airplane straight and level. The airplane could only be flown by putting in small step inputs and releasing the stick, i.e., by flying essentially open loop. Such was the case, for example, with Configuration A(F)-5(1) with a pilot rating of 10 and a PIO rating of 6. PIO tendencies were more severe with tighter closed-loop control.

It did not appear to make much difference in pilot rating whether the delay was introduced by the feel system, actuator, or both. When the primary delay came from the actuator, both pilots usually commented that the airplane was more difficult to control and had greater PIO tendencies with the tighter control used with flying VFR.

When the lag in airplane response was increased by reducing the frequency of the feel system, both pilots commented on the slow responding characteristics of the stick. The tendency was to pump and overdrive the stick, and the configuration was generally more objectionable IFR than VFR.

When the damping of the airplane short period was decreased in going from airplane A to airplane B, there were comments on the bobble and overshoot tendencies of the configurations. This was true even with the best feel system and actuator simulated, Configuration B(F)-2(10). The bobble and PIO tendencies increased at a more rapid rate with an increase in the delay or lag introduced by the control system.

When the short-period frequency of the airplane was increased, i.e., configurations with airplane C, the airplane response was described by the pilots as quick, "touchy," "snappy," or "bobbly." These remarks were also made for the configuration with the fast feel system and actuator, Configuration C(F)-2(10). The bobble was considered an annoying deficiency of the airplane, and indicates an airplane which has lighter damping closed loop than open loop. A deterioration in handling qualities again occurred as the lag or delay due to the control system increased. Configuration C(F)-5(1) had very strong, large amplitude, and divergent PIO tendencies. The PIO tendencies could only be avoided by freezing the stick or going open loop.

Lag was again a source of difficulty to the pilots for the landing approach configurations evaluated, airplane LA. Most of the deterioration in handling qualities was associated with a degradation of tracking and precise attitude and g control and not with a development of PIO tendencies. Configuration LA(F)-5(1), with a pilot rating of U9 to 10, was the only landing approach configuration with pronounced PIO tendencies and a very poor ILS approach. The pilots often commented on the low spring rate of the stick during landing approach, $(F_{ES}/\delta_{ES})_{SS} = 7.5$ lb/in. Although the pilots did not particularly like this low spring rate, they said it was representative for an airplane during landing approach and did not degrade landing approach handling qualities.

With one evaluation pilot (pilot H), some configurations were also evaluated with stick force gradients $(F_{ES}/n_z)_{SS}$ different than those shown in Table 3. Configuration A(F)-2(10), with no "bobble" or PIO tendencies, was improved by reducing the stick force gradient from 8–4 lb/g. The airplane was considered "snappier." If a configuration had PIO tendencies, reducing the stick force gradient increased the PIO tendencies and tended to make the oscillations divergent. With an increase in stick force gradient, the PIO did not disappear, but the oscillations were often described by the pilot as of low magnitude and zero damped rather than divergent.

V. Delay Time, Delay Parameter, and Phase Shift

Pilot comments often attributed the closed-loop control problems to the combined effects of a lag or delay followed by a rapid pitch response. The delay made the response difficult to predict and control. A delay parameter that considers these factors is therefore desirable in interpreting pilot comments and analyzing pilot rating data.

The higher-order response characteristics of the airplane to stick force inputs as expressed by Eqs. (9–11) can also be expressed as follows:

$$\frac{\alpha(s)}{F_{ES}(s)} = \frac{K_n(\alpha/F_{ES})_{SS}}{s^n + K_1 s^{n-1} + \ldots + K_{n-1}s + K_n} \quad (12)$$

$$\frac{\dot{\theta}(s)}{F_{ES}(s)} = \frac{(K_n/L_\alpha)(s + L_\alpha)[(\dot{\theta}/F_{ES})_{SS}]}{s^n + K_1 s^{n-1} + \ldots + K_{n-1}s + K_n} \quad (13)$$

$$\frac{n_z(s)}{F_{ES}(s)} = \frac{K_n(n_z/F_{ES})_{SS}}{s^n + K_1 s^{n-1} + \ldots + K_{n-1}s + K_n} \quad (14)$$

The steady-state responses are defined as follows:

$$(\alpha/F_{ES})_{SS} = [(\delta_{ES}/F_{ES})_{SS}][(\delta_e/\delta_{ES})_{SS}][M\delta_e/\omega_{SP}^2] \quad (15)$$

$$(\dot{\theta}/F_{ES})_{SS} = [(\delta_{ES}/F_{ES})_{SS}][(\delta_e/\delta_{ES})_{SS}][(M\;\; L_\alpha)/\omega_{SP}^2] \quad (16)$$

$$(n_z/F_{ES})_{SS} = [(\delta_{ES}/F_{ES})_{SS}][(\delta_e/\delta_{ES})_{SS}] \times [(V_0/g)(M_{\delta_e}L_\alpha/\omega_{SP}^2)] \quad (17)$$

In Eqs. (12–14) n is the sum of the orders of the elevator feel system and actuator, and the airplane short period. The constants $K_1, K_2, \ldots, K_{n-1}, K_n$ are determined by the order, frequency, and damping ratios of the feel system, actuator, and airplane.

If it is assumed that the higher-order responses can be reasonably well represented by an equivalent second-order response with a delay, then Eqs. (12–14) will assume the following form:

$$\frac{\alpha(s)}{F_{ES}(s)} = \frac{\omega_E^2[(\alpha/F_{ES})_{SS}](e^{-as})}{s^2 + 2\zeta_E\omega_E s + \omega_E^2} \quad (18)$$

$$\frac{\dot{\theta}(s)}{F_{ES}(s)} = \frac{(\omega_E^2/L_\alpha)(s + L_\alpha)[(\dot{\theta}/F_{ES})_{SS}](e^{-as})}{s^2 + 2\zeta_E\omega_E s + \omega_E^2} \quad (19)$$

$$\frac{n_z(s)}{F_{ES}(s)} = \frac{(\omega_E^2)[(n_z/F_{ES})_{SS}](e^{-as})}{s^2 + 2\zeta_E\omega_E s + \omega_E^2} \quad (20)$$

In Eqs. (18–20) ω_E and ζ_E are the equivalent second-order undamped frequency and damping ratio respectively, and a is the delay time in seconds.

If the responses of Eqs. (18–20) are truly equivalent to the responses of Eqs. (12–14) then

$$e^{as}[(1/\omega_E^2)s^2 + (2\zeta_E/\omega_E)s + 1] = (1/K_n)s^n + (K_1/K_n)s^{n-1} + \ldots + (K_{n-1}/K_n)s + 1 \quad (21)$$

By expanding e^{as} in a power series in s, and retaining terms in the expansion up to s^n, both sides of Eq. (21) will be a polynomial in s of degree n. It should be remembered that the expansion of e^{as} is convergent for all finite values of as.

By equating coefficients of the polynomials of degree n on both sides of Eq. (21), it is possible to obtain n equations in terms of the unknowns a, ζ_E, and ω_E. Obviously the frequency, damping ratio, and delay time of the equivalent second-order responses are not uniquely determined by these equations.

If the breakpoint frequency of the control system elements are sufficiently higher than the airplane short-period frequency, then the equivalent second-order response will be near the airplane short-period response. It will be determined primarily by the lower-order terms on both sides of Eq. (21). Equating the three lowest-order coefficients on both sides of Eq. (21), with e^{as} expanded in a power series, results in the following equations:

$$K_{n-1}/K_n = (2\zeta_E/\omega_E) + a \quad (22)$$

$$K_{n-2}/K_n = (1/\omega_E^2) + (2\zeta_E/\omega_E)a + \tfrac{1}{2}a^2 \quad (23)$$

$$K_{n-3}/K_n = (a/\omega_E^2) + \tfrac{1}{2}(2\zeta_E/\omega_E)a^2 + \tfrac{1}{6}a^3 \quad (24)$$

From these equations the frequency, damping ratio, and delay time of the second-order response, which approximates the higher-order response, can be determined.

If it is assumed that the rapidity of the response following the delay is adequately measured by the period of the equivalent second-order response, then

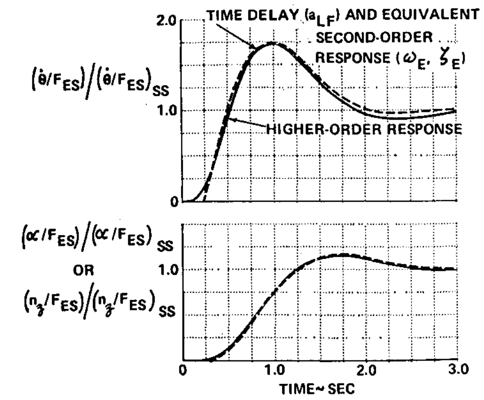

Fig. 2 Longitudinal responses for a step stick force input-configuration A(S)-2(10).

$$P_E = 2\pi/\omega_E \tag{25}$$

The ratio of the delay time to the equivalent period is a delay parameter. If the delay determined by Eqs. (22–24) is designated by a_{LF}, the delay parameter becomes

$$\text{delay parameter} = a_{LF}/P_E \tag{26}$$

If it is assumed that the equivalent second-order response is the airplane short-period response, it is possible to compute the delay time from the phase shift of the higher-order response at the airplane short-period frequency. From Eq. (21), with ω_E equal to ω_{SP}, the phase shift at the short-period frequency becomes

$$a\omega_{SP} + (\pi/2) = \phi_{CS} + (\pi/2) \tag{27}$$

The right side of Eq. (21) also contains the airplane second-order characteristic equation as a factor with a phase shift of 90° at the short-period frequency. ϕ_{CS} represents the phase shift of the control system at the short-period frequency. If the delay time determined from Eq. (27) is designated as a_ϕ, then the delay time, the period of the short period, and the delay parameter become

$$a_\phi = \phi_{CS}/\omega_{SP} \tag{28}$$

$$P_{SP} = 2\pi/\omega_{SP} \tag{29}$$

$$\text{delay parameter} = a_\phi/P_{SP} = \phi_{CS}/2\pi \tag{30}$$

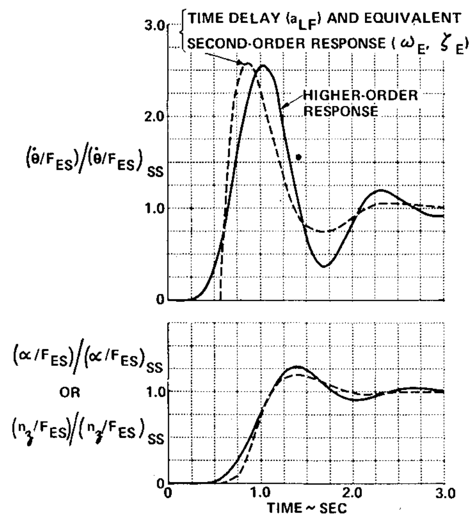

Fig. 3 Longitudinal responses for a step stick force input-configuration C(F)-5(1).

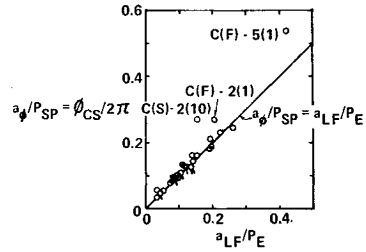

Fig. 4 Comparison of delay parameters.

Figures 2 and 3 indicate how well higher-order responses can be represented by a time delay and a second-order response whose characteristics are determined by Eqs. (22–24). In Fig. 2, the slowest element in the control system is the feel system with a frequency of 5.70 rad/sec. This frequency is approximately 2.1 times the airplane frequency, and the higher-order response is reasonably well represented by the equivalent second-order response. In Fig. 3, the slowest element in the control system is the actuator with a breakpoint frequency of 6.28 rad/sec. This frequency is quite near the airplane frequency of 5.1 rad/sec. The higher-order response is poorly approximated in this case.

Figure 4 is a comparison of delay parameters computed for all the configurations using the two methods discussed, Eqs. (26) and (30). The comparison is good when the lowest breakpoint frequency of the control system elements is sufficiently higher than the airplane short-period frequency. The delay parameter determined by Eq. (26) is generally lower than the delay parameter determined from the control system phase shift at the airplane short-period frequency. This is primarily because the equivalent frequency is always lower than the airplane frequency. The difference is quite significant when the lowest breakpoint frequency of the control system is near the airplane short-period frequency.

VI. Analysis of Pilot Ratings and PIO Ratings

The pilot ratings and PIO ratings for all the configurations evaluated are plotted as a function of the delay parameter in Figs. 5–8. Pilot B did not give the configuration a PIO rating. The desirability of a PIO rating scale was only realized at the completion of pilot B's evaluations.

The figures indicate that a strong correlation exists between pilot ratings, PIO ratings, and the delay parameter. The degradation in pilot rating with an increase in the delay parameter is due to an increase in PIO tendencies. For the landing approach configurations (Fig. 8), some of the initial degradation in pilot rating does not appear to be due to an increase in PIO tendencies. Pilot ratings and PIO ratings degrade more rapidly when the airplane damping ratio is re-

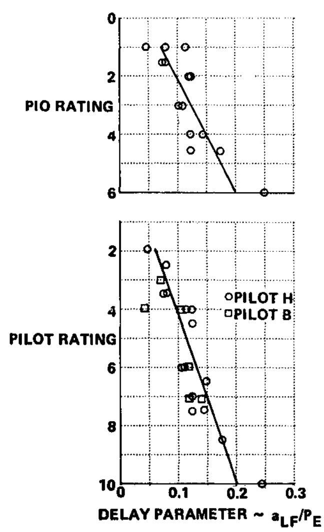

Fig. 5 Variation of pilot rating and PIO rating with delay: parameter (airplane configuration A).

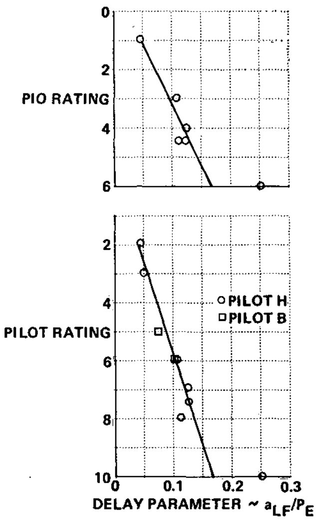

Fig. 6 Variation of pilot rating and PIO rating with delay: parameter (airplane configuration B).

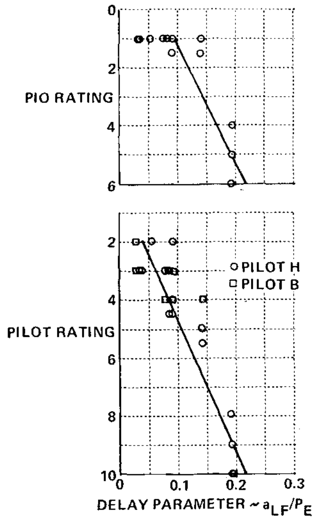

Fig. 8 Variation of pilot rating and PIO rating with delay: parameter (airplane configuration LA).

duced (Fig. 6, airplane B). The low airplane damping increases the PIO tendencies.

Based on the straight line fairing of the figures, for an airplane with a reasonably well damped short period (Figs. 5–8), the airplane changes from acceptable satisfactory to acceptable unsatisfactory at a pilot rating of 3.5, and this corresponds to a PIO rating of 1.5. The corresponding delay parameter is approximately 0.08, and the control system phase shift at the airplane short period is approximately 30°.

The airplane changes from acceptable to unacceptable with a pilot rating of 6.5. This corresponds to a PIO rating of approximately 3.5, a delay parameter between 0.14 and 0.16, and a control system phase shift of approximately 50–60°.

When the delay parameter increases to between 0.2 and 0.26, the control system phase shift increases to between 70 and 90°, the pilot rating becomes 10, and the PIO rating becomes 6. The airplane is considered uncontrollable because of the divergent PIO tendencies. Such a PIO tendency is indicated by Fig. 9, one of the many tracking records recorded in flight. The PIO occurs at approximately the airplane short-period frequency (5.1 rad/sec). The fact that phasing between stick force (F_{ES}) and the angle of attack (α), normal acceleration (n_z), and pitch rate ($\dot{\theta}$) is responsible for the tracking problems is evident from the figure. The pitch angle tracking error is shown as the angle θ_ϵ.

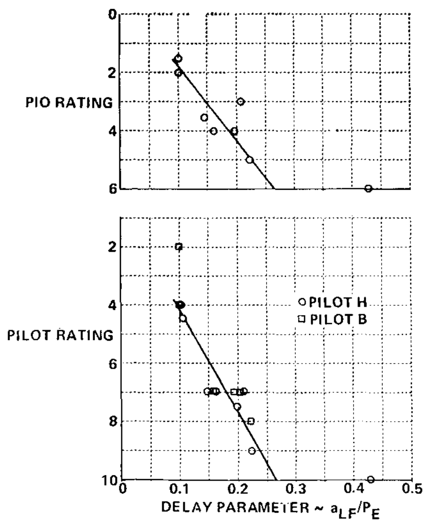

Fig. 7 Variation of pilot rating and PIO rating with delay: parameter (airplane configuration C).

VII. Comparison of Ground and In-Flight Simulation

Figure 10 compares pilot ratings in the fixed-base ground simulator[1] and in flight. There were some small differences between the ground and flight simulated configurations, but these differences are not considered significant. The numbers next to the symbols refer to the corresponding PIO ratings in flight. Configurations with little or no PIO tendencies are rated better in flight. This is not an unexpected result based on other comparisons of rating on the ground and in flight. The reverse is true of configurations with significant PIO tendencies, which are rated worse in flight. In the case of configurations with significant PIO tendencies, ground simulator results are not conservative and are actually misleading.

VIII. Conclusions

The predominant pilot comments on many of the higher-order control systems simulated were concerned with the delay following a control input and the rapidity of the response which followed the delay. Some of the configurations were considered unflyable due to large amplitude and divergent PIO tendencies. For such configurations, the pilot ratings were 10 and the PIO ratings were 6.

A strong correlation exists between pilot ratings, PIO ratings, and a delay parameter. With a degradation in control system dynamics the delay parameter increases, PIO tendencies increase, and a deterioration in handling qualities occurs. The delay parameter can also be related to the control system phase shift at the airplane short-period frequency.

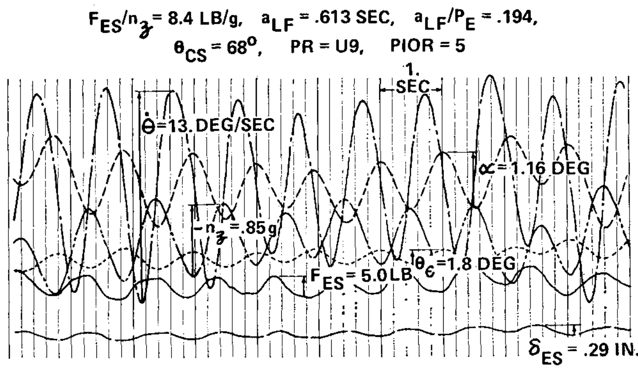

Fig. 9 IFR pitch attitude tracking task of pilot H-configuration C(F)-5(2.5).

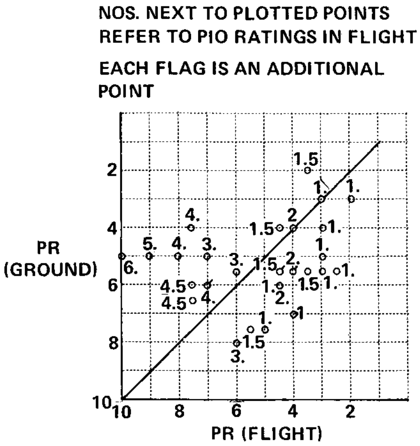

Fig. 10 Comparison of pilot ratings in flight and in a fixed base ground simulator—pilot H.

A comparison of fixed-base ground simulator vs flight evaluations indicates that configurations with significant PIO tendencies are rated poorer in flight than on the ground. In evaluating PIO tendencies, ground simulator results are not conservative and can be very misleading.

References

[1] Parrag, M. L., "Pilot Evaluation in a Ground Simulator of the Effects of Elevator Control System Dynamics in Fighter Aircraft," AFFDL-TR-67-19, Sept. 1967, Cornell Aeronautical Lab., Buffalo, N. Y.

[2] Key, D. L., "A Functional Description and Working Data for the Variable Stability System T-33 Airplane," Rept. TC-1921-F-2, Oct. 1965, Cornell Aeronautical Lab., Buffalo, N. Y.

[3] Chalk, C. R., "Simulator Investigation of the Effects of L_α and True Speed on Longitudinal Handling Qualities," *Journal of Aircraft*, Vol. 1, No. 6, Nov.-Dec. 1964, pp. 335–344.

[4] Bihrle, W., Jr., "A Handling Qualities Theory for Precise Flight Path Control," AFFDL-TR-65-198, June 1966, Wantagh, N. Y.

[5] DiFranco, D. A., "Flight Investigation of Longitudinal Short Period Frequency Requirements and PIO Tendencies," AFFDL-TR-66-163, April 1967, Cornell Aeronautical Lab., Buffalo, N. Y.

[6] DiFranco, D. A., "In-Flight Investigation of the Effects of Higher-Order Control System Dynamics on Longitudinal Handling Qualities," AFFDL-TR-68-90, July 1968, Cornell Aeronautical Lab., Buffalo, N. Y.

[7] Harper, R. P., Jr. and Cooper, G. E., "A Revised Pilot Rating Scale for the Evaluation of Handling Qualities," AGARD C.P. 17, Sept. 20–23, 1966, Compilation of papers presented to AGARD Flight Mechanics Panel Specialists' Meeting, Cambridge, England.

SEPT.-OCT. 1970 J. AIRCRAFT VOL. 7, NO. 9

Altitude Stability in Supersonic Cruising Flight

Robert F. Stengel*
Massachusetts Institute of Technology, Cambridge, Mass.

The importance of maintaining a fixed altitude is increased for aircraft cruising at supersonic speed. The difficulties of altitude control are enhanced by the small flight-path angle required to cause large vertical rates and by the time scale and basic instability of the long-period motion. The causes of altitude deviation, the magnitude of their effects, and the stability of the motion are defined for a wide range of altitudes and Mach numbers and for aircraft of varying lift-drag ratio, wing loading, thrust law, and pitch dynamics. In addition to suggesting necessary and favorable control laws, the effects of horizontal and vertical wind, atmospheric state variation, and engine "unstarts" are treated.

Nomenclature

$\mathbf{A}$ = coefficient matrix of the homogeneous equation
a = sound speed, fps
a_i = characteristic equation coefficient
AR = wing aspect ratio
$\mathbf{B}$ = scaling matrix for control inputs
$\mathbf{C}$ = scaling matrix for disturbance inputs
c = mean aerodynamic chord, ft
C_D = drag coefficient (additional subscript denotes a partial derivative)
C_L = lift coefficient (additional subscript denotes a partial derivative)
C_m = moment coefficient (additional subscript denotes a partial derivative)
DT_u = $g[2 - \nu' - M^2/(M^2 - 1)]/U(L/D)$
D_α = $2gC_{L\alpha}/\pi eAR$
D_δ = $gC_{D\delta}/C_L$
D_ξ = drag scaling coefficient for general disturbance
e = efficiency factor for drag-due-to-lift
G = compensation transfer function
g = acceleration-due-to-gravity, fps²
j = $(-1)^{1/2}$
k = radius of gyration, ft
L/D = lift-drag ratio
L_h = $g[a_h/a(1 - 1/M^2) - \beta]$
L_p = g/p
L_u = $g[2 - M^2/(M^2 - 1)]/U$
L_α = $gC_{L\alpha}/C_L$
L_δ = $gC_{L\delta}/C_L$
L_ξ = lift scaling coefficient for general disturbance
L_ρ = g/ρ
M = Mach number
m = mass, slugs
M_h = $T_h r/k^2$ (aerodynamic and aeroelastic contributions not included explicitly)
M_p = $gr/pk^2(L/D)$
M_{Th} = $gr/k^2(L/D)$

Received July 7, 1969; presented as Paper 69-813 at the AIAA Aircraft Design and Operations Meeting, July 14–16, 1969, Los Angeles, Calif.; revision received January 7, 1970.

* Member of the Research Staff, Charles Stark Draper Laboratory. Member AIAA.